# Every Child Mathematically Proficient

## An Action Plan of the Learning First Alliance

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## Foreword

This action paper was adopted by the Learning First Alliance, an organization of 12 leading national education associations. It has been informed by many distinguished experts in mathematics and mathematics education. We are pleased to acknowledge the assistance of John Dossey, Illinois State University, and Edward Esty, consultant, as well as the advice provided by Richard Askey, University of Wisconsin-Madison; Hyman Bass, Columbia University; Gail Burrill, National Council of Teachers of Mathematics; Gilberto Cuevas, University of Miami; Joan Ferrini-Mundy, Mathematical Sciences Education Board; Harold (Bud) Hodgkinson, Honorary Member, Learning First Alliance; Vinetta Jones, College Board; Andrew Porter, University of Wisconsin-Madison; Ralph Raimi, University of Rochester; Edward Silver, University of Pittsburgh; Thomas Romberg, University of Wisconsin-Madison; Irvin Vance, Michigan State University; and Judith Wurtzel, U.S. Department of Education. Although many individuals have offered suggestions that have been incorporated herein, the paper does not necessarily represent the views of any individual who assisted in the writing or provided advice and comment.

## Executive Summary

The knowledge and use of mathematics is essential in our lives, and, therefore, learning mathematics is central to elementary and secondary education. American students, however, continue to lag behind their world peers in mathematics achievement. Furthermore, substantial achievement gaps exist between groups of our students. Together, we have a major challenge to raise achievement throughout our nation. The Learning First Alliance, therefore, advances this Action Plan to bring American students to world class levels in mathematics.

**Goal.** Virtually all students starting school this fall will complete a challenging, coherent, and focused K-12 mathematics curriculum that includes core concepts of Algebra and Geometry early enough and with progressively increasing depth so that the content covered in current Algebra I and Geometry courses is mastered by the end of grade nine. Mastery of these subjects in the middle grades is essential for access to the higher levels of mathematics necessary for admission to college or good jobs after high school.

**Achievement.** Although the National Assessment of Educational Progress (NAEP) shows that U.S. students have made steady progress in mathematics achievement since 1973, the national assessment also exposes areas of weakness that must be remedied, including large racial/ethnic and urban/suburban differences in performance. Evidence from the Third International Mathematics and Science Study (TIMSS) indicates that U.S. students are performing well below their international peers from the nations with which the United States economically competes.

**Curriculum changes.** Curricular and assessment changes are needed to clearly indicate standards of what students should know and be able to do. These expectations should be held for all students, not just those who are thought to be "good at math." All students should master the content currently included in the two one-year Algebra I and Geometry courses by the end of ninth grade, with the curriculum restructured to prepare children for this coursework. Only 25 percent of U.S. ninth graders now do so.

Action steps include working on curriculum reform, providing tools to states and school districts to help them select school mathematics programs consistent with proven practice, working with educators and policymakers to improve access to high quality mathematics instruction for students from disadvantaged groups, seeking agreement on appropriate use of technology in mathematics education, and urging states and school districts to eliminate dead-end tracks in the curriculum.

**Professional development.** All students of mathematics should be taught by teachers who have been well prepared in the content of mathematics and techniques of teaching mathematics. In particular, all mathematics teachers of grades five through nine should be prepared with a solid grounding in the coursework of grades K-12 and the teaching of middle grades mathematics. Less than half of grade eight teachers have taken the mathematics courses needed as background for teaching effectively at this level. Less than 10 percent of grades five through nine teachers are mathematics specialists. Current in-service professional development programs do not generally address this lack of preparation.

Action steps include working with states, mathematics organizations, and teacher preparation programs to establish certification standards and professional development programs for teachers of mathematics in grades five through nine; developing criteria for in-service professional development; encouraging states to increase the overall requirements for teaching mathematics; creating a closer link between research findings and professional development; and working with business and industry councils to establish support for the ongoing professional development of teachers at the local level.

**Parents and the public.** Parents and community leaders must be encouraged to become involved in efforts to improve mathematics achievement. Parents and other adults can support children's performance by encouraging them, helping on out-of-class projects, discussing their homework, and helping them see the importance of mathematics to their futures. Businesses can help schools inform parents, community members, and students about the needs in mathematics for employment. They should support mathematics achievement by expanding the resources of schools and providing expertise, materials, and equipment to assist with the teaching and learning of mathematics.

Action steps include encouraging schools to discuss the mathematics curriculum with parents and others in easy-to-understand terms and to engage parents and the public in support of children's achievement; working with the media to build awareness of the importance of mathematics education; and engaging with key partners, such as the collegiate and business communities.

**Research-based reforms.** Reliance on both tradition and educational fads has played too big a role in mathematics education. Large scale change of practice must be based on research evidence that the change will lead to better student achievement.

Action steps include more research to test instructional ideas in the classroom and to compare the relative effectiveness of various instructional approaches and methods of assessment. They also include establishing methods of examining and comparing the relative effectiveness of curricular materials, and sharing this information with states and local school districts; encouraging educators and policymakers to consider research results in decisions to adopt curricula; participating in efforts to develop a research agenda for mathematics education; and continuing to engage technology in the mathematics curriculum.

## The Learning First Alliance Action Plan for Mathematics

### I. Quality Mathematics for All Students: The Needed Change

Mathematics is universally accepted as a core subject, occupying a central place in education. It is taught from the earliest grades, and provides a foundation for the learning of science and technology, as well as for the interpretation of quantitative information in other subjects. Mathematics teaches students how to reason logically and helps them develop skills that they can carry into other disciplines and many situations in real life. When taught well, mathematics can engage and delight students, helping them understand how the world works while exposing them to some of its unanswered mysteries.

In a practical sense, mathematics skills and understanding are of vital importance in determining the future success of today's young people. But unfortunately, their mathematics curriculum in too many cases does not prepare them for what the future will demand. It is impossible to determine when children are in school what level of mathematics will be required for their eventual careers. But we do know that if they do not take the mathematics courses that are the prerequisites to higher level mathematics, many opportunities will be

closed to them. Many students do not discover until too late that mathematics is an essential subject for college admission, and they have not taken the needed coursework. In addition, many disadvantaged high school students are placed in the "general curriculum" which offers a variety of oversimplified, disconnected mathematics courses and prepares them for neither college nor the world of work.

Although scores have increased for fourth, eighth, and twelfth grades on the NAEP mathematics section since 1990, American students still do not rank well compared with students of other nations in most international studies of mathematics achievement. Fortunately, much can be done to improve mathematics achievement if those who provide, govern, and work to improve mathematics education bring new energy and resources to address key points of leverage. This paper discusses four objectives for the improvement of mathematics achievement for elementary and secondary students. In brief, these objectives, which are discussed in detail in Part IV, are:

A. All our nation's students, regardless of where they live or their economic or racial and ethnic backgrounds, should have the opportunity to complete a challenging course of mathematics study that is consistent with specific benchmarks, including Algebra and Geometry by the end of the ninth grade.

B. Students must be taught by teachers who have a strong command of the subject and the best ways to teach it, which will require changes in pre-service teacher education, increased entry requirements for the initial education of teachers, and continued professional development of teachers throughout the full range of their careers.

C. Parents and teachers must be brought into the process of change in school mathematics, including discussions of curricular goals, how teaching and assessments have changed in mathematics classrooms, and how they may help improve student achievement.

D. Finally, programs of research on curricular materials, student learning, and teaching of school mathematics should be expanded. More support should be given to the translation of findings from such research into the development of high quality materials and professional development opportunities for teachers.

Change has not come far enough or fast enough to ensure that all of our 46 million public school students are afforded equal opportunity to learn everything they are capable of in school, to guarantee to the nation a well-skilled workforce, or to assure our continued economic standing in the world community. The changes that are needed cannot be accomplished by a single constituency group because they involve all parts of the education system. The Learning First Alliance represents more than 10 million individuals engaged in providing, governing, and improving America's public schools at the local, state, and national levels. It is the only national coalition focused on improving elementary and secondary education in public schools to involve both the CEO's and elected leadership of the major national organizations representing parents, teachers, curriculum specialists, school principals, administrators, school boards, state boards of education, chief state school officers, and education schools, colleges, and departments. Through this paper, the Alliance proposes a coordinated plan for improving the education of our youth in mathematics.

## II. Setting Our Goal--Guiding the Change

Our goal is for virtually all students to successfully complete a challenging K-12 mathematics curriculum that includes mastery of the content included in the two one-year Algebra I and Geometry courses by the end of grade nine.[1]

We will strive to see that virtually all students starting school in 1998 successfully complete this coursework, and will work to benefit all students now enrolled. (Currently, only 25 percent of the nation's students study Algebra I and Geometry by the end of ninth grade.) The focus of these efforts will be placed on students in the middle grades because the mathematics students study during these years has a strong effect on whether they will be able to take the higher levels of mathematics necessary for admission to college and for an increasing number of jobs, including those in rapidly expanding technical fields. However,

achievement of this goal is possible only if mathematics education from pre-kindergarten through grade four not only provides all children a strong grounding in basic arithmetic, but also lays a foundation for algebraic thinking, mathematical reasoning and geometric concepts.

**III. Changes in Student Achievement**

As one measure of progress toward our goal, we use NAEP as the best available indicator. The only continuing and nationally representative assessment currently available, NAEP is given on a regular schedule to a random sample of U.S. fourth, eighth, and twelfth grade school children, most recently in 1996. NAEP reports on four levels of achievement: Advanced, Proficient, Basic, and Below Basic.[2]

Currently, in grades four, eight, and twelve, two to four percent of students score at the Advanced level, 14 to 20 percent are Proficient, and 38 to 53 percent score at the Basic level, with the specific percentage varying by grade level. Students classified as Below Basic have failed to demonstrate what the National Assessment Governing Board (NAGB) set as the basic knowledge and skills for mathematics at those grade levels. Nearly half of the students are scoring below the Basic level. We look for an increase in the percentage of students achieving at Proficient and Advanced levels and a decrease in those below the Basic level.[3]

Data from the 1996 NAEP indicate that growth has taken place since 1990 in students' knowledge and ability to use mathematics at each grade level, both overall and in each of the five mathematics content areas measured. This growth appeared both nationwide and, almost uniformly, across the states. Twenty-eight of 33 states participating in the 1990 and 1996 assessments showed significant improvement over the six-year span at grade eight. Eighteen of 40 states participating in the 1992 and 1996 assessments showed improvement over the four-year span at grade four.

There also is a NAEP long-term trend assessment in mathematics that has tracked student performance over time by using a set of questions and tasks that was first given in 1973. The long-term trend assessment devotes a much heavier emphasis to basic number facts and computation than the national NAEP assessment program does. It shows gains in mathematics for today's nine-year olds and 13-year olds over the performance of the 1973 peers. These gains are across all mathematics topics, including basic facts and skills. This improvement is encouraging, but there are also several causes for concern. First, despite the fact that students of all racial and ethnic groups have made progress over the last two decades, there continue to be large achievement gaps between white students and their peers who are Hispanic and African American. For all students to successfully complete a challenging mathematics curriculum, we must close these gaps.

Second, there is evidence that when the main NAEP items became more challenging by requiring that students figure out answers to 40 percent of the problems on their own, instead of selecting from among multiple choice responses, cohort scores showed less progress from grade four to grade eight than previous scores.

While NAEP results provide a picture of improvement over time, they also show that the existing levels of performance are far below the expectations that anyone would hold for our youth. This view is further supported by evidence from the Third International Mathematics and Science Study (TIMSS), which indicates that U.S. students are performing well below many of their international peers at both the nine- and 13-year-old levels. At the nine-year-old level, approximately fourth grade, U.S. students' performance as a national group is at the 54th percentile of the 26 participating nations. At the 13-year-old level, two-thirds of the other 40 nations participating had higher average scores than the U.S. The poor performance of U.S. eighth grade students relative to world peers as compared with the relative performance of U.S. fourth graders is cause for concern. Even more disappointing is that the U.S. ranks last in the overall increase in student performance between the two grades for the 25 countries participating at both age levels.

If U.S. scores are rising while the nation's international standing is not, it may be because the level of

mathematics knowledge of students in other countries is also rising, that we are teaching different things, or that we are teaching the same things at a different level of depth. Many theories have been offered for the difference in performance levels between grades four and eight. Some have to do with societal factors; others with curricular differences between the countries participating (countries whose students performed well at the eighth grade level tended to teach Algebra and Geometry in the middle grades), some with the results of excessive tracking, and yet others with the focal points for teaching or teacher command of the subject matter. While more research is needed to explore what actually causes the difference in performance between the two grades, it is a problem that must be addressed on a broad front. The U.S. cannot afford to allow its students to languish in a mathematical holding pattern during these important years of intellectual development. Students' failure to complete a challenging curriculum in the middle grades affects their growth in many other areas of schooling. Both those promoting reform efforts and those critical of current changes in the school mathematics curriculum agree on the need to increase emphasis on, and raise expectations for, student performance in mathematics. Further, these expectations must extend to all students and across the full span of public education.

It is important to stress that while the strategies for change in this paper are cast broadly, the problems they are intended to address are not universally present in all U.S. schools. Powerful learning environments and talented, dedicated, and knowledgeable mathematics teachers exist in the U.S. But there are many who, through no fault of their own, are ill-prepared in mathematics and many who are assigned to teach out of field. As a result, the quality of mathematics teaching is unevenly distributed across or within states, or even school districts. In allocating resources to address the problems outlined in this paper, policymakers should pay attention to these differentials and ensure that those children in most need of high quality mathematics instruction receive priority for resources.

## IV. Strategies for Change

### Curricular and Assessment Changes

*Objective: Virtually all students starting school this fall will complete a challenging, coherent, and focused K-12 mathematics curriculum that includes core concepts of Algebra and Geometry early enough and with progressively increasing depth so that the content covered in current Algebra I and Geometry courses is mastered by the end of grade nine.*

Although state standards vary considerably, most do not go far enough in specifying what students should know and be able to do at the various checkpoints in school. Teachers need specific benchmarks for student achievement to guide what they teach. They also need materials that highlight common misconceptions and show various ways to prevent or address them. In addition, teachers need sufficient mathematical knowledge to build instruction on the different ways that various students think about problems.

In addition, the number of mathematics topics covered at each grade should be reduced to allow for greater depth in the curriculum. Research into the structuring and delivery of mathematics instruction in other nations suggests that at each grade level students take up relatively fewer topics, but cover them in greater depth so there is no need to include a given topic every year through eighth grade. In comparison, many topics are repeated at many grade levels in the U.S. TIMSS found that U.S. textbooks cover a broader range of topics every year until ninth grade than 75 percent of the 43 TIMSS countries. Another indicator of our lack of focus is that in the eighth grade the five most emphasized topics in U.S. textbooks take up less than 50 percent of the content coded as compared to a TIMSS average of 75 percent of the content coded in textbooks of other countries (and near 90 percent for Japan). Even at grade four, the five most emphasized topics account for only 60 percent of content coded compared to 85 percent of the content coded for other countries.

In the U.S., the mathematics content of eighth grade is comparable to the seventh grade curriculum of other countries. U.S. eighth graders are still studying arithmetic, while their peers in other nations have moved on to more advanced mathematics. One benchmark that should be adopted in every state is substantial

treatment by the end of grade nine of the content now covered in two one-year courses in Algebra I and Geometry. This change will require states and school districts to modify the mathematics curriculum beginning in the early grades, where fundamental concepts and processes of algebra and geometry should first be introduced.

The solution to creating a more ambitious curriculum for middle grades is not simply to require students to take a standard high school Algebra course in seventh or eighth grade. Rather, the K-nine curriculum should be restructured to allow for a more coherent transition from elementary school mathematics to higher level coursework.

**In a restructured curriculum that introduces algebraic and geometric concepts early enough, by the end of ninth grade, students should be able to:**

- Understand and use formulas (expressed in words as well as symbols) and equations.
- Understand and use variables and equations to represent quantity and relationships in linear, quadratic, and exponential settings.
- Demonstrate fluency in translating among equivalent algebraic representations numerically, graphically, and symbolically, including negative and fractional components.
- Use formulas and equations to model verbally presented relationships and situations, such as rate, work, and percent mixture problems.
- Recognize, describe, extend, create, generalize, and use linear patterns, as well as some of the more common non-linear patterns such as quadratic and geometric progressions.
- Recognize problem situations in which decisions to be made involve relations among quantitative variables--one variable changing over time, or several variables changing in response to each other.
- Use numerical tables, graphs, symbolic expressions, and verbal descriptions to describe and predict rates of change and patterns of change in variables.
- Apply mathematical reasoning to the solution of equations and the evaluation of formulas, and be able to explain solutions and generalizations by means of reference to the fundamental properties of the real number system.
- Solve linear, quadratic, and systems of linear equations in two variables analytically and graphically, with and without calculators, and be able to describe the relationship between the representations.
- Use coordinate systems to analyze relationships and solve problems.
- Model and solve problems representing a wide range of complexity with algebraic representations and reasoning.

**Students should also be able to:**

- Visualize, describe, draw, and construct common plane and solid figures, detailing their relationships.
- Apply geometric relationships to phenomena in the world.
- Compare geometric objects in terms of their properties and relationships such as congruence and similarity, including how objects change under single or multiple transformations.
- Represent problem situations with geometric models and apply properties of figures in a variety of contexts to solve mathematical and commonplace problems.

- Use the Pythagorean Theorem and ratio and proportion to solve problems.

- Use spatial sense and geometric reasoning, both inductive and deductive, to form and validate conjectures, form arguments, and justify generalizations.

- Remember, use and informally prove simple theorems concerning congruence, similarity, and mensuration on lines, circles, and triangles with deductive reasoning.

- Represent geometric entities on a plane coordinate system and apply algebraic methods to their description and analysis. Additional competencies, including competencies in the areas of measurement and proportion, would be included in benchmarks for the middle grades performance.

Once states and school districts establish the mathematics curriculum as suggested above, they need to provide related professional development for teachers. Revised assessments, based on frameworks that clearly express what we expect students to know and be able to do, will also be needed to monitor student progress and diagnose student problems.

It will be important to ensure that the revised standards and related curriculum apply to all middle school students, not just those considered to be "good at math" or "college material." Children will not learn what they are not taught. Many young people will be effectively denied the opportunity to go on to college and to enter many careers unless teachers, counselors, and school administrators hold higher expectations for all students in the area of mathematics achievement, eliminate "general math" in the curriculum, and provide the additional supports some students will need to succeed in higher level mathematics courses. A disproportionate number of children who have been offered less rigorous mathematics courses and poorer quality instruction are African American or Hispanic, even after controlling for CTBS scores. The Second International Mathematics Study indicated that white students had a disproportionately higher rate of placement in Algebra classes in the schools where Algebra I was offered as a special eighth grade class. More must be done to achieve equality of opportunity for students in school mathematics classrooms nationwide. Changing these expectations is a massive task requiring the leadership of administrators and policymakers to support modifications to curricula, teacher education, and professional development.

**To achieve a more challenging mathematics curriculum for all students, we must:**

- At the state and school district level, specify clear benchmarks and provide for a more focused and challenging study of mathematics for each grade or group of grades. Teacher preparation, textbooks and other curriculum materials, assessments, and mechanisms for holding schools accountable should be aligned with these benchmarks;

- Eliminate dead-end tracks in the school curriculum such as "general mathematics;"

- At the school district level, create grade-by-grade curriculum guides consistent with state benchmarks for achievement;

- Assure that all schools have solid curricula with challenging expectations for all students, thereby eliminating differential access to high quality mathematics instruction across regions, states, or groups of students;

- Continue to study how technology should be used to further student learning in mathematics;

- Develop clear, consistent, and regularly administered assessment programs for monitoring student progress aligned with curriculum benchmarks; and

- Reach for an achievement performance profile for students on NAEP assessments that increases the percentage of students achieving at the highest levels, with virtually all students at the Basic level or above, at least 60 percent at the Proficient level, and 15 percent at the Advanced level for each of the

three grade levels sampled.[4]

**The Alliance can help achieve this objective by:**

1. Working with the National Council of Teachers of Mathematics (NCTM) and the Mathematical Sciences Education Board (MSEB) to address the issues of curricular and assessment system revision. This should include submitting comments regarding the revision of NCTM Standards to be released in the year 2000. The assessment and curriculum structure framework should be influenced by research in mathematics education including what is done in other countries with successful mathematics education programs.

2. Working with leading mathematics teachers, mathematicians, middle grade/ junior high school curricular experts, commercial publishers, and educational researchers to develop criteria for states and school districts to use in the selection of school mathematics programs for grades five through nine. This should also be done for grades K-four.

3. Working with educators and policymakers to address the questions and problems of access to quality mathematics instruction for students from disadvantaged groups.

4. Urging states and school districts to eliminate dead-end tracks in the school curriculum and ensure that all students have multiple opportunities to acquire the knowledge and skills necessary to compete in either higher education or the workplace upon graduation from high school.

**B. Professional Development of Teachers**

*Objective: All students of mathematics will be taught by teachers who have been well prepared in the content of mathematics and techniques of teaching mathematics. In particular, all mathematics teachers of grades five through nine will be mathematics specialists, educated to meet the mathematical needs of students studying a challenging curriculum that includes introductory Algebra and Geometry, and will meet the standards established through the Interstate New Teachers Assessment and Support Consortium (INTASC).*

The delivery of a challenging mathematics curriculum in schools will demand a great deal of teachers and, therefore, of those who prepare, certify, hire, and support them. It will require that teachers have strong backgrounds in the mathematics they teach and a well-developed understanding of the mathematics taught across the full K-12 span. Teachers will need knowledge of the previous grades' content to provide special help to those still working to attain deep understanding of requisite material. They will need knowledge of the following grades' content to deal with insightful student questions and to align their development and presentation of content with the future learning needs of their students. They will need to know a variety of ways to structure lessons and to balance the emphasis on learning skills, understanding concepts, and solving problems. In addition, teachers must be able to communicate enthusiasm about mathematics for its own sake, as well as the role that mathematics programs play in students' future opportunities. They should relate well to students, understand their development, and be able to communicate well with parents, to help them support their children's learning. To ensure that all students have equality of opportunity, teachers and all school personnel will need to hold high expectations for all students, not just those thought to be college-bound.

The TIMSS Video Tape Classroom Study of Japanese, German, and American mathematics classrooms provides some insights that help explain the relationship of teacher preparation and support to achievement. This study examined the methods by which teachers introduce and teach concepts and applications; how they present alternative solution methods; their use of mathematical principles, properties, and definitions; how they justify and prove the statements made; and the ways they connect lessons into coherent wholes. Findings on the U.S. eighth grade classrooms indicated that, while the teachers professed to be teaching according to NCTM standards which emphasize problem solving rather than rote learning, the teaching observed suggested this was not the case. Instead, the U.S. teachers'

objectives seemed in general to be developing a skill in the absence of meaning. In contrast, Japanese teachers were more likely to be engaged in developing their students' mathematical thinking--understanding concepts or creating alter-native solutions to the problems under discussion. U.S. students were almost never asked to provide justification or proof for their assertions, but nearly one-half of Japanese and German teachers expected this as a matter of course.

The tapes captured what was central to the study of mathematics in the three participating countries. Problem solving was at the core of Japanese classrooms, while practicing a taught algorithm or process was at the heart of most U.S. classroom lessons. While the difference in approach reflects differences in curricula discussed earlier, it also may reflect the degree of preparation and support these teachers receive. The Japanese teachers, unlike U.S. teachers, undergo long-term structured apprenticeships in their profession. They know much more mathematics than most U.S. teachers do, which enables them to work more flexibly based on students' needs. They also are provided with a great deal more assistance regarding how to teach specific lessons, and with comments on where students typically have trouble, with suggestions for what to do when the expected misconceptions arise. This help is provided for teachers at all levels.

Results of teacher surveys from NAEP and TIMSS show that the majority of U.S. teachers have far less than the minimal coursework we recommend for the teaching of mathematics, both in mathematics content and mathematics methods, particularly in the elementary and middle grades. Data from NAEP assessments and other sources show that less than three-fourths of eighth grade mathematics teachers have ever had a course in the teaching of mathematics at this level. And despite the fact that they are expected to teach more complex content, teachers in grades five through eight often have the same mathematics background as teachers in grades K-four.

A related problem is the lack of articulation in many teacher education programs. Mathematics education, mathematics, and general pedagogy courses may be spread over several departments, and the mathematics coursework itself may be taken all or partially on a community college campus totally separated from the mathematics methods segment taught as part of the final degree program. In many cases, the mathematics coursework, should any be required, is only the course required for liberal arts students. Often, the coursework has little to do with the content of the K-nine curriculum.

Teachers should be prepared to use assessment to shape instruction and improve learning opportunities for their students. They should learn to recognize and address common student problems and misconceptions. A teacher has to know enough mathematics to recognize what underlies a student's mistakes, and use that knowledge to help the student correct the mistakes and develop a fuller understanding. The teacher also should be prepared to deal with students whose understanding is more advanced. Teachers should be prepared to use varied instructional tools and classroom techniques appropriate for their students and a variety of assessments to find out what students understand.

Current in-service professional development programs generally do not address the inadequacies of pre-service teacher preparation programs. Once teachers reach the classroom, they receive little, if any, professional development in the teaching of mathematics. When teachers do receive help, it is often in the form of one-day workshops or scattered courses. In contrast to current practice, effective professional development addresses both content knowledge and the best ways to teach that content. It occurs over time, including follow-up, long-term support. Teachers in such programs are able to see effective teaching methods in action, to practice what they learn, and to learn new mathematics that will support their teaching practice. They build on their own knowledge and learn to deal with typical student misunderstandings. Finally, effective professional development is delivered by people with a reputation for their expertise in the subject matter and its delivery to students at their level.

Teacher professional development at all levels must change to reflect a new standard of excellence. Beyond initial education to become a teacher, schools and states should support and require significant professional development programs for their teachers of mathematics. Such programs should provide opportunities--both in mathematical content and pedagogy--for teachers related to the grade spans they

teach.

Because we are arguing for a new level of challenging curriculum in the middle grades, it is especially important to ensure that the mathematics teachers of grades five through nine have specialized knowledge of the content they teach. They also should be educated to understand how young adolescents learn mathematics, in particular the concepts and skills of Algebra and Geometry, and should be equipped with pedagogical expertise. While these mathematics specialists can come from either elementary or secondary teacher preparation programs, they would ideally come from middle grade preparation programs, but meet all of the standards recommended by The Mathematical Association of America (MAA) and NCTM teacher education guidelines for specialists at this level, and meet the INTASC standards.

Middle grades mathematics specialists need to have a solid grounding in all the mathematics of grades K-12. This would require collegiate mathematics coursework spanning the mathematics of the elementary grades. Coursework in content would include teaching of both Algebra and Geometry, an introduction to probability and statistics, coursework in mathematics content in the middle grades classroom, coursework on the concepts and techniques of calculus, and knowledge of the use of technology in teaching the mathematics of these grades. In addition, the teacher should have one or more courses specifically focusing on the curriculum and teaching of mathematics to students in grades five through nine. While some schools are too small to departmentalize their teaching staffs, they should see that their teachers of mathematics have achieved the equivalent of such a specialization in mathematics and have a solid knowledge of the entire K-12 mathematics curriculum.

Universities, school districts, states, and regional accrediting bodies need to develop ways of promoting high quality programs for students who are preparing to teach mathematics for these grades. States and regional accrediting bodies need to prepare special certification/licensure for such teachers, and school districts should ensure that teachers demonstrate mastery of both pedagogy and the subject matter they teach. These changes will require breaking down walls between university departments of mathematics and curriculum and instruction, between universities and community colleges, between colleges of liberal arts and science and education, between universities and state departments of instruction, and between middle school mathematics departments and their secondary counterparts. It will require teacher preparation programs, courses, and degrees that better articulate abstract mathematics with the needs of the classroom. This will, in turn, require that mathematicians work with colleges of education more closely than they do now in order to ensure that future generations continue to produce mathematicians of the highest caliber.

Ultimately, a significant barrier to achieving the objective of well-prepared teachers in the content of mathematics will be the shortage of mathematics teachers that meet the recommended criteria. One factor contributing to the teacher shortage in this subject is the significantly higher salaries available outside the teaching profession for people skilled in mathematics. Teachers in the U.S. are paid significantly less in relation to Germany and Japan.

Requiring higher levels of content knowledge, as we recommend, will make this shortage worse, although in the long run it should improve student achievement. States and school districts struggling with the issue of teacher shortages will need to develop mechanisms and explore incentives to help attract qualified individuals to the teaching of mathematics, particularly to those schools serving low-income children. Initial thoughts might focus on financial incentives to assist current teachers to add mathematics to their certifications. Or qualified math teachers might volunteer to take extra student loads, rather than leave them to be taught by those who are unqualified. Serious thought also should be given to attracting individuals with strong mathematical backgrounds into teaching, without ignoring the need for pedagogical as well as mathematical expertise.

Although this section emphasizes the importance of well-prepared teachers, it is important to stress that administrators and policymakers have a critical role in improving student achievement. These individuals make key decisions on resources available for professional development--and must ensure that these learning opportunities are of high quality. They are responsible for ensuring that the school offers high

quality mathematics programs and has adequate and appropriate texts, enrichment materials, and supplies. Administrators are responsible for providing safe and orderly classrooms with uninterrupted instructional time conducive to teaching and learning.

### To enable all students to learn from well-prepared mathematics teachers, we must:

- Bring all pre-service teacher education programs into line with the MAA, INTASC and NCTM standards for what teachers should know about mathematics and mathematics education;
- Develop, support, and require teacher professional development in mathematics and mathematics education over the full span of teaching careers, with special emphasis on the first five years of induction into the profession and on continued growth in teaching mathematics;
- Create a closer link between professional development programs and research in the teaching and learning of mathematics in the middle grades;
- Equip teachers with tools and supports to enable them to help children of all backgrounds complete a challenging mathematics curriculum;
- Enforce teacher licensure and certification requirements in school settings to recognize specialist teachers and to ensure that students have access to fully qualified teachers of mathematics at all levels; and
- Explore incentives to attract qualified individuals to mathematics teaching.

### The Alliance can help address this objective by:

1. Developing a common understanding among Alliance organizations about what constitutes quality professional development supported by research on teacher learning.
2. Working with the National Partnership for Excellence and Accountability in Teaching on a national conference focusing on effective professional development for mathematics teachers.
3. Exploring ways in which the Alliance can work with the National Council for Accreditation of Teacher Education to bring all teacher education programs into line with The Mathematical Association of America's and National Council of Teachers of Mathematics' standards for what mathematics and mathematics-specific pedagogy teachers at each level should know.
4. Working with states, mathematics organizations, and universities with teacher preparation programs to establish certification standards, induction programs, and support preparation programs for grades five through nine mathematics specialists.
5. Working with the U.S. Department of Education and the National Science Foundation to develop recommended criteria for in-service professional development and to establish professional development programs for teachers currently teaching in all grades, but especially in grades five through nine so that their students can benefit from better-prepared teachers.
6. Encouraging teachers to raise their expectations for student performance in mathematics, especially focusing on their expectations for students from underrepresented or disadvantaged groups.
7. Encouraging administrators to play a leadership role in supporting the hiring of skilled mathematics teachers, to ensure that children with the greatest needs are taught by the most effective teachers, and to support high quality professional development that is informed by research and aligned with the curriculum.

**C. Public Awareness and Support**

*Objective: Build strong levels of public support for a challenging mathematics curriculum at all levels of*

*schooling, K-12.*

Parents, business, and community leaders must join educators in encouraging students to pursue higher levels of mathematics achievement. According to an Alliance-sponsored poll, one in four adults say they have or had a "fear of math," or that they feel unable to assist an eighth grader with mathematics homework. It is important that these adults not pass on their fears to children, and that they recognize that regardless of their own experience with school mathematics, they can support children's performance by encouraging them, monitoring their progress, helping on out-of-class projects, and discussing their homework. They can help children develop a love of mathematics, and they can help them appreciate that mathematics achievement is important to their future.

It is especially important for members of economically disadvantaged communities to know that low-income students who go on to college are much more likely than other students to have taken Algebra and Geometry early. In one representative national sample, only 26 percent of low-income students who did not take Geometry went to college, compared with 71 percent of low-income students who did take Geometry. In addition, a growing number of careers require high levels of mathematics. According to the National Skill Standards Board, 11 of 12 industry sectors say students need algebra, geometry, or trigonometry. Nearly 40 percent of all 17-year-olds do not have the necessary math skills to hold down a production job in manufacturing.

Effective administrative leadership encourages teachers, students, and parents to recognize the importance of a quality mathematics program and actively support its implementation. Parents and interested community members should be informed about the specific academic standards that children are to meet at each grade level. They should be offered the opportunity to be involved in efforts to change mathematics education. They also should hold schools accountable for making it possible for all students to complete a challenging mathematics curriculum.

To further this effort, schools and community groups should hold special sessions for parents and others to discuss the nature and shape of the mathematics curriculum and the importance of mathematics education to future success. This information should be supported by high quality materials that lay out what children should be able to do at different grade levels (including actual examples of mathematics problems), illustrate the relevance of challenging mathematics to success in college and a wide range of careers, and suggest roles for community members, including business and adult volunteers.

Employers can help schools inform parents, community members, and students about the role mathematics plays in a variety of fields by participating in school-to-work programs, mathematics fairs, career days, and other activities. They can articulate workplace academic skill requirements, support professional development for teachers that helps them connect mathematics to the world of work, and encourage their employees who are parents to increase their involvement with the school. They can also support improved mathematics achievement by expanding the resources of schools and providing expertise, materials, and equipment to assist with the teaching and learning of mathematics. Finally, they can underscore the importance of student achievement by requesting student records of prospective employees.

**To achieve this objective, schools must:**

- Engage parents and other adults in the community to support children's study and performance;
- Discuss the mathematics curriculum with parents and other interested community members and encourage them to participate in activities that affect the teaching and learning of mathematics;
- Interpret assessment results for parents, community leaders, and the media, noting the relationship of the data to local goals and local trends over time; and
- Involve the business community in activities that will improve the school mathematics curriculum and

increase opportunities for teachers.

**The Alliance can address this objective by:**

1. Encouraging the education community to reach out to parents, students, and community members in support of improved mathematics achievement, particularly among students in grades five through nine and focusing on low-income communities.

2. Identifying or producing high-quality materials that articulate the goals of the curriculum to parents and other interested community members in easy-to-understand terms, with examples showing why the content is important for their students.

3. Working with the media to build an awareness of the importance of mathematics achievement.

4. Engaging with key partners, such as the professional mathematics associations and business community, in this effort.

**D. Research and Development**

**Objective: Convincing research is available and will inform practice and materials adoption.**

Reliance on both tradition and educational fads has played too big a role in mathematics education. There must, of course, be a place for the investigation of new ideas since the fields of mathematics and cognitive science are dynamic and continually yield new knowledge that may have implications for mathematics teaching and learning. Large scale change of practice, however, should not be based on the publication of a new research paper or a current enthusiasm, but should occur when a body of sound research convinces us that a change will lead to better student achievement. For research to be convincing, we need cumulative studies of the results of mathematics education research. The production of such cumulative studies should be a high priority so that educators, like doctors, have easy access to validation studies and other papers to guide their practice. In the absence of cumulative, convincing research, recommendations for change should be tempered by common sense.

More research is needed to test instructional ideas in the classroom and to compare the relative effectiveness of various instructional approaches and methods of assessment in promoting student learning. This is especially needed given findings from TIMSS and the Survey of Mathematics and Science Opportunities. They show that multiple approaches to teaching mathematics, used in different high-achieving countries, are effective in helping students learn to high levels. In addition, a study of high-achieving schools in Texas--selected because they were situated in areas where students were considered "at-risk"--reveals that these schools used a wide variety of materials and approaches. Research is needed to identify what conditions and support are needed in each case for an approach to be effective.

Although several new mathematics curriculum programs are emerging at the middle-school level, we need continuing research on these programs and others. More research also is needed to support the development of high quality material that reflects solid programs of mathematics that are supported by research and practice.

**To achieve this objective, schools, states, and federal agencies must:**

- Bring research into the public schools to help understand the problems preventing student performance from reaching desired levels;

- Carefully evaluate the relative effectiveness of varied approaches to achieving standards for school mathematics for students, for teachers, and for instructional programs as a whole;

- Encourage inclusion of teachers in all facets of research, from question formulation to implementation;

- Continue to monitor national and international achievement and curricular trends to provide a base for comparison and targets for improvement;

- Implement and evaluate strategies to achieve equity for all students in access to both quality curricula and teachers who demonstrate proficiency in mathematics and how to teach it; and

- Translate research findings into strategies to improve the effectiveness of various instructional approaches, commercial and project materials, and the use of technology to foster student achievement and increase rates of student retention in school mathematics programs.

**The Alliance can address this objective by:**

1. Working with the NSF and U.S. Department of Education to assure continued funding of large scale assessments so the data relevant to the recommended curricular changes can continue to be monitored from national (NAEP) and international (TIMSS-like) levels.

2. Establishing methods of examining and comparing the relative effectiveness of curricular materials in lab-like settings before they are used in the classroom, and sharing this information with states and local school districts.

3. Encouraging educators and policymakers to consider research perspectives, methodologies, and results in decisions to adopt curricula.

4. Participating in efforts to develop a research agenda for mathematics education.

5. Continuing to study the appropriate role that technology should play in the mathematics curriculum.

## Endnotes

1. Middle school students must be well grounded in five major areas (Number Sense, Properties and Operations; Measurement; Data Analysis, Statistics, and Probability; Algebra and Functions; and Geometry and Spatial Sense). The Alliance is focussing on Algebra and Geometry because they are powerful gatekeepers for access to post-secondary education studies and key jobs. "Mastery of the content included in the two one-year Alegebra I and Geometry courses" is not intended to imply a preference for the existing course structure. A fuller discussion of the recommended topics to be covered is outlined in Part IV.

2. In 1990 the National Assessment Governing Board (NAGB) established the first three levels of achievement. The setting of expectations was started in 1990 as the NAEP focused more and aligned with the recommendations of the National Council of Teachers of Mathematics (NCTM) Standards. Based on this experience, NAGB made adjustments in the content expectations assigned to each of the levels in 1992. This set of expectations is currently in effect for the NAEP assessments. The long-term trend assessment items were not affected by these changes.

3. To provide a sense of expectation for student performance, if we assume the same achievement levels are used in the future, we would set targets that virtually all students would be at or above the Basic level and that there would be substantial increases in the students performing at the Proficient and Advanced levels, with specific targets of 60 percent Proficient and 15 percent Advanced. However, there is controversy regarding whether the current levels of achievement used by NAGB are appropriate. There will continue to be analysis about whether these levels should be used. If the content used for determining mathematics achievement changes, the achievement levels and specific targets would presumably also need to be changed.

4. See footnote number 3.

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